Lecture 1

INTRODUCTION

The course "Fundamentals of circuit theory" is based on the theory of electricity, which includes two inseparable components: electrical engineering and electronics.

Electrical engineering studies high power objects, where energy problems are a matter of great interest, while electronics deals with low power objects, where information problems are the most interesting. Fundamentals of electrical engineering and radio electronics rely on the use of electrical energy — a unique type of energy, which has two main benefits:

1) ease of long-distance transmission, which is used in high-voltage transmission lines that transmit direct and alternating current at a distance of hundreds and thousands of kilometers;

2) ease of conversion into other energy forms, such as conversion of electrical energy into mechanical energy in electric motors, conversion of electrical energy into thermal energy — in electro thermal furnaces and electrical heaters, conversion of electrical energy into light — in lighting devices.

One can hardly imagine modern life without the use of electricity: from household needs to the most advanced achievements of scientific and technological advance. For example, where in the early days of aviation the power of on board electrical equipment did not exceed fractions of kilowatts, now on large airliners of the An-255 "Mria" (Dream) type it is measured in hundreds of kilowatts. Space exploration cannot be imagined without the use of electricity and radio either.

The beginning of electrical science dates to XVI century, when the English scholar William Gilbert (1544–1603) wrote his treatise "On the magnet, the magnetic body and a big magnet — the Earth".

The first works in the field of electricity in Russia belong to the brilliant Russian scientist M. V. Lomonosov. Systematic studies of electrical phenomena were initiated by T. Oersted, a Dane, who discovered the magnetic effect of electric current, by the British researchers Faraday and Joule who discovered the law of electromagnetic induction and thermal effects of electric current respectively, by the Russian scientist V. V. Petrov who first observed the electrical arc, and by E. Lenz who, simultaneously with Joule, established the correlation between the amount of heat produced by electric current in a conductor and the value of this current.

The use of electrical phenomena in telecommunications dates back to 1832 when the Russian scientist P. Shilling created the first model of electromagnetic telegraph. Since 1876, when A. Bell demonstrated the first electromagnetic telephone in the world, electricity has been used in telephonic communication.

In 1873, in his treatise on electricity and electromagnetism the English scientist J. Maxwell formulated the fundamental equations of classical electrodynamics, from which followed the existence of electromagnetic waves, which was experimentally proved by the German scientist H. Herz in 1895.

But neither J. Maxwell nor H. Herts considered practical application of this discovery possible. Only in 1895 the Russian scientist A. S. Popov invented the world's first radio transmitter and receiver. It is the point from which electronics as an area of science and technology that deals with the study and application of electromagnetic waves begins.

Issues to be considered in electrical engineering and radio electronics can be studied both in terms of electric circuit theory (ECT) and in terms of electromagnetic field theory (EFT). ECT deals with the concepts of voltage and current described by systems of ordinary differential equations. EFT deals with the concepts of electric and magnetic fields that are described by systems of differential equations in partial derivatives.

The main problems of ECT are analysis and synthesis. By analysis we mean the study of electromagnetic processes occurring in a given circuit, the structure of the circuit, its parameters and input stimuli being known. Synthesis is an inverse problem where the structure, parameters and input stimuli are to be determined, the characteristics of the electromagnetic processes in the circuit being known. It follows from the definition that synthesis is more complex and ambiguous task.

This book deals with the analysis of linear circuits only and only in stationary mode.

The basics of modern methods of analyzing circuits were laid by Kirchhoff, who in 1847 published the basic laws of electric circuits. Kirchhoff's laws were complemented by Maxwell, who suggested the method of node voltages. Further, the theory of analysis developed in the direction of procedure formalization. In 1925 Franklin formulated Kirchhoff's laws in matrix form. Seven years later, Foster, developing the theory of linear graphs for the analysis of electric circuits, derived a number of topological formulas.

Nowadays ECT is enjoying a certain rise. This is mainly due to the fact that both circuits and their functions are growing more and more complicated. Besides, ECT methods are very useful in the studies of various processes of non-electric nature (e.g. physical, chemical, economic, etc.) because they can be modeled by electric circuits with this or that degree of precision.

The complexity of analysis and synthesis of complex circuits is significantly reduced with the use of electronic computers (EC).On the other hand, computer-based calculation gives a powerful impetus to further development of ECT as it stimulates improvement of classical methods of circuit analysis and synthesis and development of new targeted automation and computer applications and methods.

Wide application of computers for circuit analysis imposes high requirements on engineering education making it necessary to master computer methods of circuit analysis and synthesis.

1. ELECTRIC CIRCUITS

1.1. Basic Concepts of Circuit Theory

The content of the course "Fundamentals of Circuit theory (FCT)" is based on the application of physical laws and an extensive use of mathematical methods [1–3].

The subject of circuit theory (CT) is the development of engineering methods for studying processes in electric and radio electronic devices.

The principal method of CT is the substitution of simplified models (the processes in which are described by scalar values — currents and voltages) for real devices.

An electric circuit is a set of devices and objects that make a path for an electric current. Electromagnetic processes in an electric circuit can be described by means of the concepts of electromotive force (EMF), voltage and current.

Conventional graphic representation of an electric circuit is called a circuit diagram.

A schematic circuit diagram shows all electric circuit elements and connections between them using conventional symbols.

Electric current is an ordered motion of charge carriers (electrons, ions). It is assumed that the positive current direction is the direction in which positive charge carriers move. The conventional positive direction of current can be chosen arbitrarily. The amount of current is defined by the following relationship:

$$i = \frac{dq}{dt}, \qquad (1.1)$$

where i is the instantaneous current value, measured in amperes (A); q, t are the charge and time, respectively, measured in coulombs (C) and seconds (s).

Hence, we see that electric current can be direct and alternating.

The electromotive force is defined as the work of extraneous forces, required to move a single positive charge inside an electric energy source from the terminal of lowest potential to the terminal of highest potential.

When there are no other sources in the circuit, the positive direction of the EMF is assumed to be the direction that coincides with the current

direction across the energy source. The instantaneous EMF value is designated "e" and measured in volts (V).

The potential is the work required to move a single positive charge between a given point in an electric field and a point where the potential is assumed to be equal to zero, for instance — an infinitely distant point. It is designated φ and is measured in volts.

The voltage between the two points A and B in an electric field is the work required to move a single positive charge between the given points in the field. It is designated "u". It is, also, obvious that the voltage between the points A and B in the electric field is the potential difference between these points.

$$u_{AB} = \varphi_A - \varphi_B. \qquad (1.2)$$

Voltage is quantified by the relationship:

$$u = \frac{dw}{dq}, \qquad (1.3)$$

where u is the value of instantaneous voltage, and w is the energy of the electric field, measured in joules (j).

The positive direction of current is assumed to point in the positive direction of voltage across the sections of the circuit that contain no energy sources. The voltage direction in circuit sections that contain energy sources is opposite to the direction of EMF.

The work of electric field is the change of electric field energy. From (1.1) and (1.3) it follows that:

$$dw = udq = iudt. \qquad (1.4)$$

The total work that characterizes a change in the electric field energy at the instant of time t is defined by the relationship:

$$w(t) = \int_{-\infty}^{t} iud\tau, \qquad (1.5)$$

where τ is the current time value, $-\infty < \tau \le t$.

If at any instant $w(t) \ge 0$ in a circuit section under consideration, it means that the section is an electric load, and is called "passive". Even, if at any instant of time $w(t) < 0$, it means that this section is a source of electric energy, and is called "active".

Power is the rate of electric field energy change.

Quantitatively, power is defined by the relationship:

$$p = \frac{dw}{dt}, \tag{1.6}$$

where p is the instantaneous power value, measured in watts (W).

From (1.4) and (1.6)

$$p = iu. \tag{1.7}$$

If at a given instant of time $p > 0$, that is — the current and voltage directions in a circuit section coincide, it means that the section is dissipating electric energy. If $p < 0$, then the circuit section is supplying electric energy. Let us use (1.7) in (1.5) and obtain:

$$w(t) = \int_{-\infty}^{t} p\,d\tau. \tag{1.8}$$

It is obvious that the energy supplied to the circuit during the time period of $\Delta t = t_2 - t_1$ is defined by the expression:

$$W = w(t_1) - w(t_2) = \int_{t_1}^{t_2} p\,dt.$$

Issues covered in the theory of electrical circuits and electromagnetic field theory, presented in Fig. 1.1

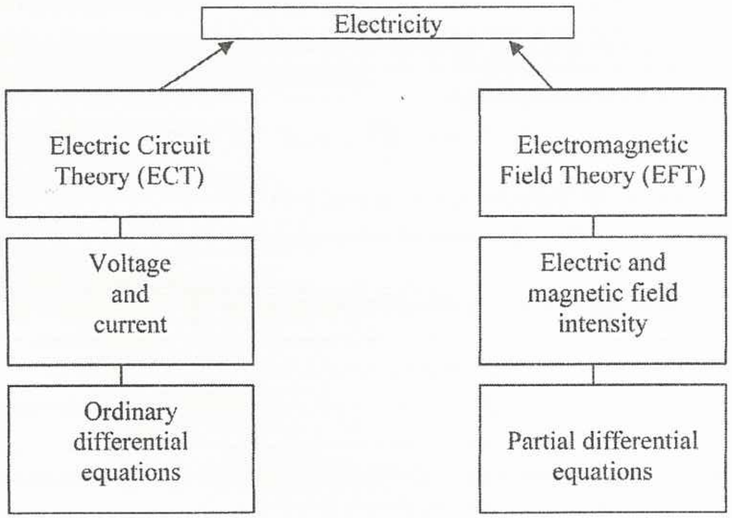


Fig. 1.1

1.2. Electric Circuit Elements

In an electric circuit, we distinguish passive and active elements. Passive elements dissipate (absorb) electric energy, for them $p > 0$, while active elements supply (generate) electric energy, for them $p < 0$.

The passive elements include resistance, inductance, and capacitance.

The active elements are energy sources (voltage and current sources).

Let us consider the passive elements of an electric circuit.

Resistance is an idealized passive element in which electric energy is converted irreversibly into any other kind of energy (thermal, mechanical, or light).

Physically, as a real electric circuit element, it takes the form of a resistor. Its schematic representation is shown in Fig. 1.2, *a*.

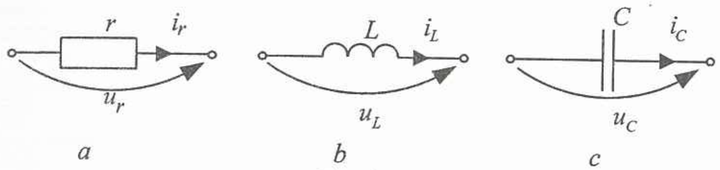


Fig. 1.2

Generally, the resistor *voltage—current characteristic* (VCC) $u = f(i)$ is nonlinear (Fig. 1.3, *a*)

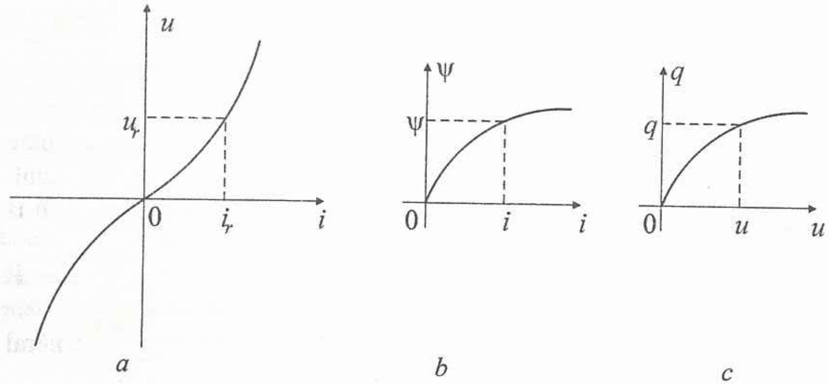


Fig. 1.3

The static resistance of a resistor is:

$$r_{st} = \frac{u_r}{i_r}. \tag{1.9}$$

Dynamic resistance is:

$$r_{dyn} = \frac{du}{di}.$$

Resistance is measured in Ohms.

If the VCC is linear, the resistor is called a linear resistor; otherwise it is called a non-linear resistor.

Expression (1.9) represents Ohm's law. It can be written in the following way:

$$i_r = \frac{1}{r_{st}} u_r = g_{st} u_r.$$

Here,

$$g_{st} = \frac{1}{r_{st}},$$

conductance is measured in Siemens (S).

Since the resistance and conductance dissipate electric energy irreversibly, they are called dissipative elements.

The instantaneous power of resistance is defined in the following way:

$$p_r = i_r u_r = i_r^2 r = u_r^2 g.$$

Electric energy in a resistance is always non-negative

$$w_r = \int_{-\infty}^{t} p_r d\tau = r \int_{-\infty}^{t} i_r^2 d\tau = g \int_{-\infty}^{t} u_r^2 d\tau \geq 0.$$

This means that a resistance at any instant of time can only dissipate electric energy. Thus, it follows that the resistance is a passive element. The inductance is an idealized passive element the property of which is not to dissipate electric energy but to store it in the magnetic field.

Physically, as an electric circuit element, it is the inductance coil. It is shown schematically in Fig. 1.2, *b*.

The flux-current characteristic of a coil $\psi = f(i)$, in the general case, is non-linear (Fig. 1.3, *b*). The static inductance of a coil is:

$$L_{st} = \frac{\psi}{i}, \qquad (1.10)$$

where ψ is the magnetic-flux interlinkage of self-induction, measured in *Webers* (*Wb*).

Dynamic inductance is:

$$L_{dyn} = \frac{d\psi}{di}.$$

Inductance is measured in *Henry* (H).

If the flux-current characteristic is linear, the inductance is called linear, otherwise — nonlinear.

According to the law of induction, the voltage across an inductance is the following:

$$u_L = \frac{d\psi}{dt}.$$

Taking into account (1.10), we obtain:

$$u_L = L\frac{di}{dt}.$$

Hence,

$$i_L = \frac{1}{L}\int_{-\infty}^{t} u_L d\tau = \frac{1}{L}\left(\int_{-\infty}^{0} u_L d\tau + \int_{0}^{t} u_L d\tau\right) = i_L(0) + \frac{1}{L}\int_{0}^{t} u_L d\tau.$$

Here,

$$i_L(0) = \frac{1}{L}\int_{-\infty}^{0} u_L d\tau,$$

is the current in the inductance at the instant of time $t = 0$.

Instantaneous inductance power is

$$p_L = i_L u_L = L i_L \frac{di_L}{dt}.$$

If u_L and i_L have like signs, the inductance is charged, and the energy goes into the coil magnetic field. If u_L and i_L have unlike signs, the inductance is discharged, and the coil magnetic field gives the energy back into the external circuit.

The magnetic energy stored in an inductance is always non-negative.

$$w_L = \int_{-\infty}^{t} p_L d\tau = \int_{-\infty}^{t} L i_L \frac{di_L}{d\tau} d\tau = L\int_{0}^{i_L} i_L di_L = \frac{Li_L^2}{2} = \frac{\psi^2}{2L} \geq 0. \qquad (1.11)$$

That is, the inductance, by definition, is a passive element.

The capacitance is an idealized passive element the property of which is not to absorb electric energy, but to store it in the electric field.

Physically, it takes the form of the real electric circuit element called the capacitor. It is shown schematically in Fig. 1.2, *b*. The coulomb — the charge-voltage characteristic of a capacitor $q = f(u)$, in the general case, is non-linear (Fig. 1.3, *b*). Static capacitance is:

$$C_{st} = \frac{q}{u}. \qquad (1.12)$$

Dynamic capacitance is:

$$C_{dyn} = \frac{dq}{du}.$$

Capacitance is measured in farads (F).

If the charge-voltage characteristic is linear, the capacitance is called linear, otherwise — non-linear.

The current in a capacitance is:

$$i_C = \frac{dq}{dt}.$$

Taking into account (1.12), we obtain

$$i_C = C\frac{du_C}{dt}.$$

Hence,

$$u_C = \frac{1}{C}\int_{-\infty}^{t} i_C d\tau = \frac{1}{C}\int_{-\infty}^{0} i_C d\tau + \frac{1}{C}\int_{0}^{t} i_C d\tau =$$

$$= u_C(0) + \frac{1}{C}\int_{0}^{t} i_C d\tau.$$

Here,

$$u_C(0) = \frac{1}{C}\int_{-\infty}^{0} i_C d\tau,$$

is the voltage across the capacitance at the instant of time $t = 0$.

Instantaneous capacitance power is:

$$p_C = \frac{dw}{dt} = u_C i_C = Cu_C\frac{du_C}{dt}.$$

If u_C and i_C have like signs, the capacitance is charged, the energy goes to the capacitor electric field. If u_C and i_C have unlike signs the capacitance is discharged, the capacitor electric field gives the energy back to the external circuit.

The electric energy stored in a capacitance is always non-negative

$$W_C = \int_{-\infty}^{t} p_C d\tau = \int_{-\infty}^{t} Cu_C\frac{du_C}{d\tau}d\tau =$$

$$= C\int_{0}^{u_C} u_C du_C = \frac{Cu_C^2}{2} = \frac{q^2}{2C} \geq 0. \qquad (1.13)$$

This means that, the capacitance, by definition, is a passive element.

Let us consider the active elements of the electric circuit.

The voltage source. There are ideal and real voltage sources.

The ideal voltage source is an idealized active element, the voltage across the terminals of which does not depend on the current flowing through it.

The voltage across the terminals of an ideal voltage source is equal to the EMF *e*. Schematically, the ideal voltage source is shown in Fig. 1.4, *a*.

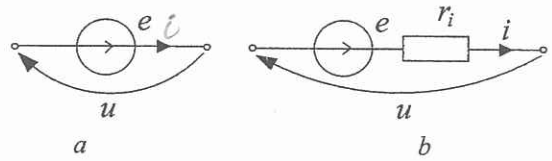


Fig. 1.4

The volt-ampere (external) characteristic $u = f(i)$ of the ideal voltage source has the form of a straight line parallel to the current axis (Fig. 1.5, full line). We can see that, when the current i increases, the power $p = i \cdot u$ rises without bound. Therefore, the ideal voltage source is called the source of infinite power.

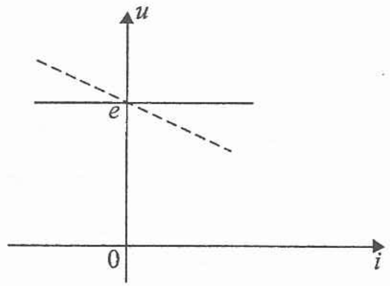


Fig. 1.5

A real voltage source can be represented in the form of a series connection of an ideal voltage source e and the internal resistance r_i (Fig. 1.4, *b*). The volt-ampere characteristic of the real voltage source is as follows:

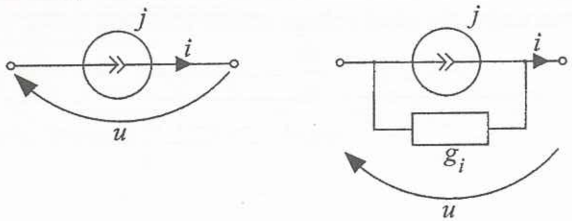
$$u = e - ir_i.$$

On a graph, this characteristic is represented as an inclined straight line (Fig. 1.5, dotted line). We can see that when the current i increases, the voltage u drops, that is the power $p = iu$ cannot be infinite. Therefore, the real voltage source is called the source of final power.

The current source. There are ideal and real current sources.

The ideal current source is an idealized active element, the current of which does not depend on the voltage across its terminals.

Schematically, the ideal current source is shown in Fig. 1.6, *a*.



a *b*

Fig. 1.6

The volt-ampere characteristic $u = f(i)$ has the form of a straight line parallel to the voltage axis (Fig. 1.7, full line). We can see that when the voltage u increases, the power $p = iu$ rises without bound. Therefore, the ideal current source is called the source of infinite power.

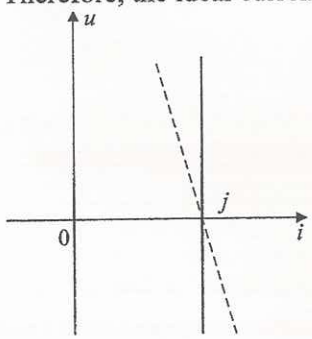


Fig. 1.7

A real current source can be represented in the form of a parallel connection of the ideal current source j and the internal conduction g_i (Fig. 1.6, *b*). The volt-ampere characteristic of a real current source is:

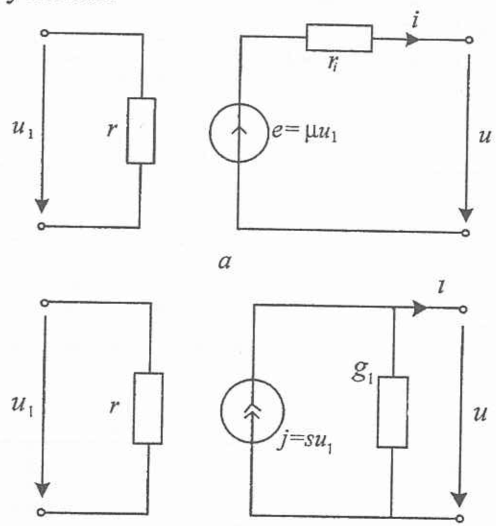
$$i = j - ug_i. \qquad (1.14)$$

On the graph, this characteristic is represented by an inclined straight line (Fig. 1.7, dotted line). We can see that when the voltage u increases, the current i

decreases, that is, the power $p = iu$ can not be infinite. Therefore, the real current source is called the source of final power.

The voltage and current sources considered are called "independent sources" (self-contained). In circuit theory, there are also dependent (non-self-contained) sources, i.e., those in which the EMF e or the current j depend on the voltage or current in any circuit section.

Their diagrams are shown in Fig. 1.8. Here, the EMF e of the voltage source (Fig. 1.8, *a*) depends on the voltage u_1 across any circuit section. The current j of the current source (Fig. 1.8, *b*) depends also on the voltage u_1 across any circuit section. In Fig. 1.8, the values μ and s are proportionality factors.



b

Fig. 1.8

1.3. Dual Elements and Circuits

Let us put the relationships for the electric circuit elements obtained earlier into Table 1.1. It is obvious from Table 1.1 that the expressions, which correspond to the pairs of resistance and conduction, inductance and capacitance, voltage source and current source, are of similar structure. If, in the expressions for resistance, inductance, and voltage source, we substitute u for i, i for u, g for r, C for L, j for e — we will obtain the relationships for conductance, capacitance, and current source, respectively.

Table 1.1

Element type	Basic relationships			
	current	voltage	power	energy
Resistance	$i=\frac{u}{r}$	$u=ir$	$p=i^2r$	$w=\int_{-\infty}^{t} i^2 r d\tau$
Conductance	$i=ug$	$u=\frac{i}{g}$	$p=u^2g$	$w=\int_{-\infty}^{t} ugd\tau$
Inductance	$i=i(0)+\frac{1}{L}\int_{-\infty}^{t} ud\tau$	$u=L\frac{di}{dt}$	$p=Li\frac{di}{dt}$	$w=\frac{Li^2}{2}$
Capacitance	$i=C\frac{du}{dt}$	$u=u(0)+\frac{1}{C}\int_{-\infty}^{t} id\tau$	$p=Cu\frac{du}{dt}$	$w=\frac{Cu^2}{2}$
Voltage source	$i=\frac{e-u}{r_i}$	$u=e-ir_i$	$p=iu$	$w=\int_{-\infty}^{t} iud\tau$
Current source	$i=j-ug_i$	$u=\frac{j-i}{g_i}$	$p=ui$	$w=\int_{-\infty}^{t} uid\tau$

The elements of an electric circuit, for which the main relationships have the same structure and can be obtained one from the other by means of substitution, are called dual. It is not only elements that can be dual, but circuits as well. Examples of dual circuits are given in Fig. 1.9. Here, the series connection of the voltage source e and the elements r, L, C with the voltages u_r, u_L, u_C and the current i (Fig. 1.9, *a*) is replaced, in the dual circuit, by the parallel connection of the current source j and elements g, C, L with currents i_g, i_C, i_L and the voltage u (Fig. 1.9, *b*).

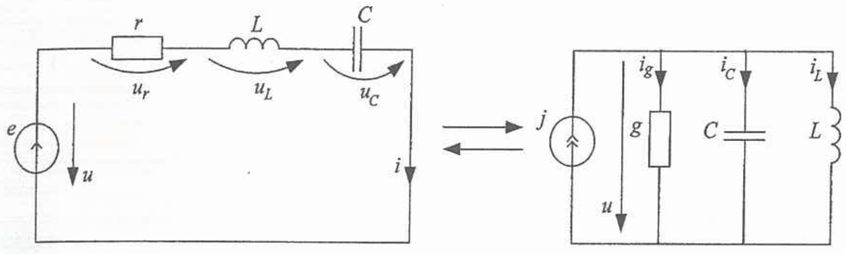


Fig. 1.9

Thus, we obtain the dual conception of electric circuits: parallel connection — series connection. Table 1.2 also includes the dual circuit conception and relationships considered above.

Table 1.2

Conceptions and Relationships	
Initial Circuit	Dual Circuit
Series Connection	Parallel Connection
Kirchhoff's current law (KCL) $\sum_k i_k=0$	Kirchhoff's voltage law (KVL) $\sum_k u_k=0$
Series connection of elements $r_e=\sum_k r_k$	Parallel connection of elements $g_e=\sum_k g_k$
$L_e=\sum_k L_k$	$C_e=\sum_k C_k$
$\frac{1}{C_e}=\sum_k \frac{1}{C_k}$	$\frac{1}{L_e}=\sum_k \frac{1}{L_k}$
Mesh current method	Node voltage method
Constant-current generator method	Constant-voltage generator method

Example

The current in an inductance is changed in accordance with the sinusoidal law

$$i(t) = I_m \sin \omega t, \quad t > 0$$

and is equal to zero at $t < 0$. Find the law of voltage variation on the terminals of the inductance, the instantaneous power and energy. Build time diagrams for the data obtained.

Solution

The voltage across the inductance is determined by the expression

$$u(t) = L\frac{di_L(t)}{dt}.$$

The derivative of the current with respect to time

$$\frac{di_L(t)}{dt} = \frac{d[I_m \sin \omega t]}{dt} = \omega I_m \cos \omega t.$$

The voltage across the inductance

$$u_L(t) = \omega L I_m \cos \omega t.$$

Instantaneous power

$$p(t) = i_L(t) \cdot u_L(t) = I_m \sin \omega t \cdot \omega L I_m \cos \omega t = 0,5\omega L I_m^2 \sin 2\omega t.$$

Instantaneous energy

$$\omega(t) = 0,5 L i_L^2(t) = 0,5 L I_m^2 \sin^2 \omega t =$$
$$= 0,25 L I_m^2 (1 - \cos 2\omega t) = 0,25 L I_m^2 - 0,25 L I_m^2 \cos 2\omega t.$$

Fig. 1.10 shows the time diagrams of current, voltage, power and energy. It is seen that the instantaneous power changes the sign; it means that in a certain period of time the inductance consumes energy from the circuit, and in another — it gives the energy to the circuit. Energy is always greater than or equal to zero, thus the inductance is a passive element.

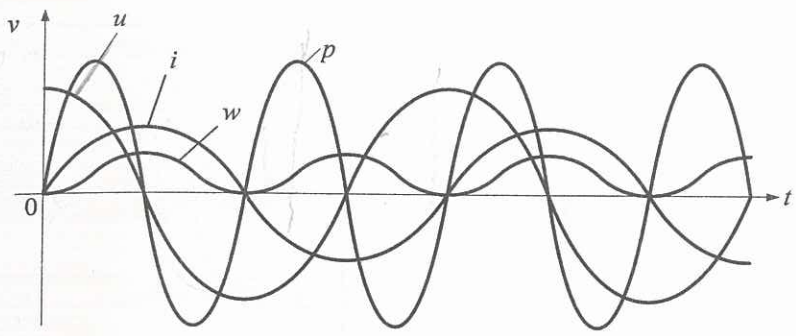


Fig. 1.10